

Environment assisted entanglement enhancement

Biplab Ghosh, A. S. Majumdar,* and N. Nayak

S. N. Bose National Centre for Basic Sciences, Salt Lake, Kolkata 700 098, India

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We consider various dissipative atom-cavity systems and show that their collective dynamics can be used to maximize entanglement for intermediate values of the cavity leakage parameter κ . We first consider the interaction of a single two-level atom with one of two coupled microwave cavities and show analytically that the atom-cavity entanglement increases with cavity leakage. We next consider a system of two atoms passing successively through a cavity and derive the expression for the maximum value of κ in terms of the Rabi angle gt , for which the two-atom entanglement can be increased. Finally, numerical investigation of micromaser dynamics also reveals the increase of two-atom entanglement with stronger cavity-environment coupling for experimentally attainable values of the micromaser parameters.

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Quantum entanglement has emerged as a highly useful resource in communication and computation protocols[1] in recent years. There have been many proposals for generating atomic entanglement and entanglement between cavity modes through atom-photon interactions[2], and some notable experimental demonstrations have also been performed[3]. Several interesting features and applications of entanglement can be obtained in devices involving optical and microwave cavities[4]. In all these schemes interaction with the surrounding heat bath and cavity leakage has to be monitored such that rapid decoherence[5] is unable to destroy the created entanglement within the time-frame required for observation.

Though most proposals of entanglement generation rely on methods to reduce the coupling with the environment, there have been some suggestions[6, 7, 8, 9, 10] for creating entanglement between two or more parties by their collective interactions with a common environment. The effectiveness of collective interactions in the dynamics of quantum optical systems has been appreciated much earlier[11]. Specific examples of environment induced entanglement have been worked out recently. Schemes of using the decay of the cavity field to induce atom-atom or cavity-cavity entanglement have been proposed[6]. Braun has shown that entanglement can be created between two qubits which do not interact directly with each other, but interact with a common heat bath[7]. A corollary of this result is the mediation of atomic entanglement by a thermal field inside a cavity[8]. Other proposals involving the collective dynamics of trapped ions, squeezed and thermal fields, and quantum-reservoir engineering have also been suggested[9, 10]. The purpose of the present work is to show not just the creation of entanglement with environmental assistance, but to demonstrate the feasibility of actual enhancement of entanglement in real workable devices.

To this end in this Letter we show that entanglement in atom-cavity devices can be quantitatively increased

by increasing the cavity damping rate. We illustrate our point by first considering two examples of tripartite systems ((i) two cavities and a single atom, and (ii) one cavity and two atoms) where we obtain analytically the expressions for the atom-cavity and two-atom concurrences, respectively, as functions of the cavity leakage parameter κ . It is seen explicitly that the concurrences are maximised for intermediate values of κ/g (where g is the atom-cavity coupling constant). Plenio and Hulega[10] have earlier shown numerically that entanglement between two optical cavities driven by an external optical white noise field can be maximised for intermediate values of the cavity damping rates. In this work we derive analytically a similar result, using simple but practically realizable systems with two examples.

We next consider the experimentally workable micromaser[12]. The micromaser device is well known for its utility in the generation of entangled atomic states[13]. The controlled monitoring of dissipative effects makes it possible to study fundamental aspects like nonlocality and information transfer through the micromaser[14]. Many of these features have been demonstrated in several experiments performed using the micromaser[12]. Here we choose certain experimentally achieved range of values for the micromaser parameters[12] and show through numerical analysis that the entanglement between a pair of atoms can be increased with the increase of cavity damping κ up to a certain range of its values.

A. A two-level atom interacting with one of two maximally entangled cavities

We first consider two initially maximally entangled single-mode cavities (C_1 and C_2). [Such a system can be prepared by sending a single circular Rydberg atom in its excited state through two identical and initially empty high-Q microwave cavities[15].] A two-level Rydberg atom A_1 prepared in the ground state $|g\rangle$ passes through the cavity C_1 . The resonant interaction between

the two-level atom and cavity mode frequency takes place with the Rabi angle gt . In presence of the cavity dissipation the dynamics of the flight of the atom is governed by the evolution equation

$$\dot{\rho} = \dot{\rho}|_{\text{atom-field}} + \dot{\rho}|_{\text{field-reservoir}}, \quad (1)$$

At temperature $T = 0$ K the average thermal photon number is zero. Since we are working with a two-level Rydberg atom, its lifetime is much larger compared to the atom-cavity interaction time and hence we can neglect the atomic dissipation. The first term on the r.h.s. of Eq.(1) evolves under the usual Jaynes-Cummings interaction, and the second term is given by

$$\dot{\rho}|_{\text{field-reservoir}} = -\kappa(a^\dagger a \rho - 2a \rho a^\dagger + \rho a^\dagger a). \quad (2)$$

where κ is the cavity leakage constant. We consider the approximation of a two-level cavity, i.e., the probability of getting a two- or more than two-photon number state of the cavities is zero. Under the secular approximation[16] the time-evolved density matrix of the reduced state of the first cavity and the atom ($\rho(t)_{C_1 C_2 A_1}$) is given by

$$\rho(t)_{C_1 A_1} = \text{Tr}_{C_2}(\rho_{C_1 C_2 A_1}) = \begin{pmatrix} \alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_3 & -\alpha_4 & 0 \\ 0 & \alpha_4 & \alpha_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (3)$$

in the basis $|0_1 g_1\rangle$, $|0_1 e_1\rangle$, $|1_1 g_1\rangle$, and $|1_1 e_1\rangle$, where $|0\rangle$ and $|1\rangle$ represent the 0-photon and the 1-photon number basis states of the cavities, respectively, and α_i are given by

$$\begin{aligned} \alpha_1 &= \left(1 - \frac{e^{-\kappa_1 t}}{2}\right)e^{-2\kappa_2 t}, \\ \alpha_2 &= (\cos^2 gt)e^{-\kappa_1 t}\left(1 - \frac{e^{-2\kappa_2 t}}{2}\right), \\ \alpha_3 &= (\sin^2 gt)e^{-\kappa_1 t}\left(1 - \frac{e^{-2\kappa_2 t}}{2}\right), \\ \alpha_4 &= i(\sin 2gt)e^{-\kappa_1 t}\left(1 - \frac{e^{-2\kappa_2 t}}{2}\right), \end{aligned} \quad (4)$$

where κ_1 and κ_2 are the cavity leakage constants of C_1 and C_2 , respectively.

We quantify the entanglement using the measure *concurrence*, which for a general state ρ_{12} is defined as[17] $C(\rho_{12}) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})$, where the λ_i are the eigenvalues of $\rho_{12}(\sigma_y \otimes \sigma_y)\rho_{12}^*(\sigma_y \otimes \sigma_y)$ in descending order. We compute the concurrence for $\rho(t)_{C_1 A_1}$, which is given by

$$C(\rho(t)_{C_1 A_1}) = 2 \cos gt \sin gte^{-\kappa_1 t} \left(1 - \frac{e^{-2\kappa_2 t}}{2}\right) \quad (5)$$

If we take $\kappa_1 = \kappa_2 = \kappa$, one gets

$$C(\rho(t)_{C_1 A_1}) = 2C_{ideal}(e^{-\kappa t} - e^{-3\kappa t}/2) \quad (6)$$

For small κ , we can write

$$C(\rho(t)_{C_1 A_1}) \approx C_{ideal}(1 + \kappa t). \quad (7)$$

where $C_{ideal} = \cos gt \sin gt$ is the value of the concurrence in the case of ideal cavities with no dissipation. It follows from Eq.(5) that for large κ one gets $C(\rho(t)_{C_1 A_1}) \rightarrow 0$, as expected. This feature of dissipation assisted increase of entanglement is observed for a range of values of κ , and the maximum of concurrence is obtained for the value of $\kappa/g = [\ln(3/2)]/(2gt)$ at fixed gt . As an aside, it is interesting to note that the entanglement between the two cavities C_1 and C_2 falls off with increasing κ , thus providing a manifestation of the monogamous nature of entanglement between the pairs $C_1 A_1$ and $C_1 C_2$. A monogamy inequality[18] for this situation has been verified[19]. The increase of entanglement with κ as seen in Eq.(7) follows from the collective nature of the dynamics of the two cavities, as is apparent from the structure of the elements of the atom-cavity state given in Eqs.(4) where the α_i 's are the sums of two terms involving κ_1 and κ_2 respectively. This motivates us to look for similar collective effects in other simple tripartite systems such as the one involving the interaction of a single cavity with two successive atoms, considered below.

B. A single cavity and two two-level atoms

Before considering the real micromaser, we now investigate a micromaser-type system where two two-level atoms, the first prepared in the excited state $|e\rangle$, and the second prepared in the ground state $|g\rangle$, are sent into a vacuum cavity one after the other, i.e., there is no spatial overlap between the two atoms. Our purpose here is to demonstrate analytically the increase of two-atom entanglement in this model with the increase of cavity damping rate. We compute the time-evolved density state for the tripartite system of the two atoms and the cavity under the same secular approximation, and the approximation of a two level (zero or one photon) cavity. The reduced density state of the pair of atoms $A_1 A_2$ is given by

$$\rho(t)_{A_1 A_2} = \text{Tr}_{C_1}(\rho_{A_1 A_2 C_1}) = \begin{pmatrix} \gamma_2 & 0 & 0 & 0 \\ 0 & \gamma_3 & -\gamma_4 & 0 \\ 0 & -\gamma_4 & \gamma_1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (8)$$

in the basis of $|g_1 g_2\rangle$, $|g_1 e_2\rangle$, $|e_1 g_2\rangle$, and $|e_1 e_2\rangle$ states, and where γ_i are given by

$$\begin{aligned} \gamma_1 &= (1 - \sin^2(gt)e^{-\kappa t}), \\ \gamma_2 &= \cos^2(gt)\sin^2(gt)e^{-2\kappa t}, \\ \gamma_3 &= \sin^4(gt)e^{-2\kappa t}, \\ \gamma_4 &= \left(\sin(gt)e^{-\kappa t/2} - \frac{\kappa}{2g}\cos(gt)e^{-\kappa t/2} + \frac{\kappa}{2g}\right) \times \\ &\quad \cos(gt)\sin(gt)e^{-\kappa t}, \end{aligned} \quad (9)$$

The concurrence for the joint two-atom state $\rho(t)_{A_1 A_2}$ is given by

$$C(\rho(t)_{A_1 A_2}) = 2 \sin^2 gt e^{-\kappa t} \sqrt{(1 - \sin^2(gt) e^{-\kappa t})} (10)$$

For values of κ/g and gt such that $(\tan^2 gt)(gt)(\kappa/g) \ll 1$, we can write

$$C(\rho(t)_{A_1 A_2}) \approx C_{ideal}(1 + \frac{1}{2} \kappa t \tan^2(gt) - \kappa t). \quad (11)$$

where C_{ideal} (no dissipation) in this case is given by $C_{ideal} = 2 \cos gt \sin^2 gt$. Thus, enhancement of $C(\rho(t)_{A_1 A_2})$, i.e., the increase of the atom-atom mixed-state entanglement over its value in the ideal cavity case is possible if we choose interaction time judiciously, such that $\tan(gt) > \sqrt{2}$. For fixed gt , the concurrence $C(\rho(t)_{A_1 A_2})$ is maximised at $\kappa/g = [\ln((3/2) \sin^2(gt))]/(gt)$. Further increase of cavity damping κ , causes the two-atom entanglement to fall off. This sets the stage for us to investigate next the full micromaser dynamics.

C. The one-atom micromaser

We now consider the real micromaser which has been experimentally operational[12]. The mathematical model for the micromaser has been well studied, and is described in detail in [14, 20, 21]. Its essential features are outlined briefly here. The cavity is pumped to its steady state by a Poissonian stream of atoms passing through it one at a time, with the time of flight through the cavity t being the same for every atom. The dynamics of these individual flights are governed by the evolution equation with three kinds of interactions given by

$$\dot{\rho} = \dot{\rho}|_{atom-reservoir} + \dot{\rho}|_{field-reservoir} + \dot{\rho}|_{atom-field} (12)$$

where the strength of three couplings are given by the parameters Γ (the atomic dissipation constant), κ (the cavity leakage constant), and g (the atom-field interaction constant) with the individual expressions provided in [14, 20]. Obviously, $\Gamma = 0 = g$ describes the dynamics of the cavity when there is no atom inside it. The finite temperature of the cavity is represented by the average thermal photons n_{th} , obtained from B-E statistics.

The density matrix of the steady-state cavity field ρ_f^{ss} can be obtained by solving the above equation and tracing over the reservoir and atomic variables[14, 20]. The photon distribution function is then given by

$$P_n^{ss} = \langle n | \rho_f^{ss} | n \rangle \quad (13)$$

We display the steady state photon statistics for the micromaser for experimentally realizable values[12] of the parameters N ($N = R/2\kappa$, where R denotes the number of atoms passing through the cavity per second), n_{th} and

TABLE I: Steady state photon statistics for the micromaser with the parameter values $n_{th} = 0.033$, $N = 1$, and $gt = 3\pi/4$.

κ/g	P_0	P_1	P_2	$\langle n \rangle$
0.1	0.771	0.220	0.007	0.236
0.01	0.664	0.316	0.014	0.359
0.005	0.655	0.324	0.015	0.370
0.0000807	0.645	0.332	0.016	0.382

gt in Table 1. The probability of getting two (P_2) or more photons inside the cavity is negligible. The photon statistics thus provides a justification for our earlier assumption of a two-level cavity (P_0 and P_1) used for obtaining our analytical results of entanglement enhancement through dissipation in the previous examples. However, our present analysis for the real micromaser does not employ this assumption.

We compute the atomic entanglement generated between two experimental atoms that pass successively through the micromaser cavity. The tripartite joint state of the cavity and the two atoms is obtained by summing over all n . The reduced density state of the two atoms after passing through the cavity field is given by

$$\rho(t)_{A_1 A_2} = \text{tr}_f(\rho(t)_{A_1 A_2 C_1}) = \begin{pmatrix} \beta_5 & 0 & 0 & 0 \\ 0 & \beta_3 & \beta_4 & 0 \\ 0 & \beta_4 & \beta_2 & 0 \\ 0 & 0 & 0 & \beta_1 \end{pmatrix} (14)$$

where the β_i are given by

$$\begin{aligned} \beta_1 &= \sum_n P_n^{ss} \cos^4(\sqrt{n+1}gt), \\ \beta_2 &= \sum_n P_n^{ss} \cos^2(\sqrt{n+1}gt) \sin^2(\sqrt{n+1}gt), \\ \beta_3 &= \sum_n P_n^{ss} \cos^2(\sqrt{n+2}gt) \sin^2(\sqrt{n+1}gt), \\ \beta_4 &= \sum_n P_n^{ss} \sin^2(\sqrt{n+1}gt) \cos(\sqrt{n+1}gt) \cos(\sqrt{n+2}gt), \\ \beta_5 &= \sum_n P_n^{ss} \sin^2(\sqrt{n+1}gt) \sin^2(\sqrt{n+2}gt). \end{aligned} \quad (15)$$

The concurrence of the two-atom state is plotted with respect to the Rabi angle gt in Fig. 1 choosing the cavity temperature as in the operational micromaser[12]. We see that the entanglement between the two atoms increases as we increase the cavity dissipation parameter κ/g from the experimental values (solid curves in Fig. 1) at a fixed values of the Rabi angle gt . Increased damping of the micromaser cavity causes the average cavity photon number $\langle n \rangle$ to go down, as displayed in Table 1. The collective dynamics of the system causes the magnitude of two-atom entanglement to rise with decreasing $\langle n \rangle$. This anti-correlation of the two-atom entanglement with the cavity photon number $\langle n \rangle$ has also

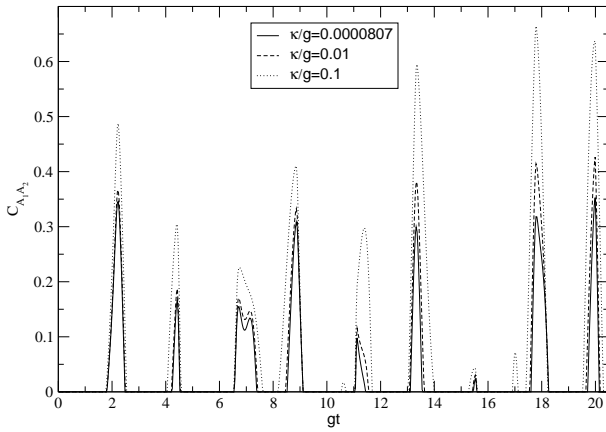


FIG. 1: The two-atom concurrence $C_{A_1 A_2}$ is plotted versus the Rabi angle gt for various values of the cavity leakage parameter κ . Here we choose $N = 1$ and $n_{th} = 0.033$

been observed in earlier works[22]. It is expected that further increase of κ/g beyond the values shown in the figure would make the concurrence to fall. However, the validity of the micromaser theory that we have used[20] is itself limited to low dissipation values by the secular approximation[16].

To summarize, in this Letter we have presented concrete examples of the increase of entanglement caused by the stronger interaction of a part of a composite system with its environment. We have first considered the atom-cavity entanglement in a system comprised of two entangled cavities and a two-level atom[15]. The derived expression for the atom-cavity concurrence clearly shows the maximization of entanglement for intermediate values of the cavity damping. A similar analytical result is also obtained for two-atom entanglement in the case of a micromaser-type system involving a single cavity and two atoms. We have further investigated a model for the real micromaser[12] and demonstrated the increase of atomic entanglement with cavity damping for fixed atom-cavity interaction times for experimentally operational values of the micromaser parameters. With further development, it may be possible to utilize this effect of environment assisted entanglement enhancement in information processing involving multipartite systems where the interactions times may not be easily controllable. In conclusion, we highlight that atom-cavity systems provide much scope for the quantitative tests of several manifestations of environment induced entanglement[6, 7, 8] in experimentally realizable situations.

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* Corresponding author email:archan@bose.res.in

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